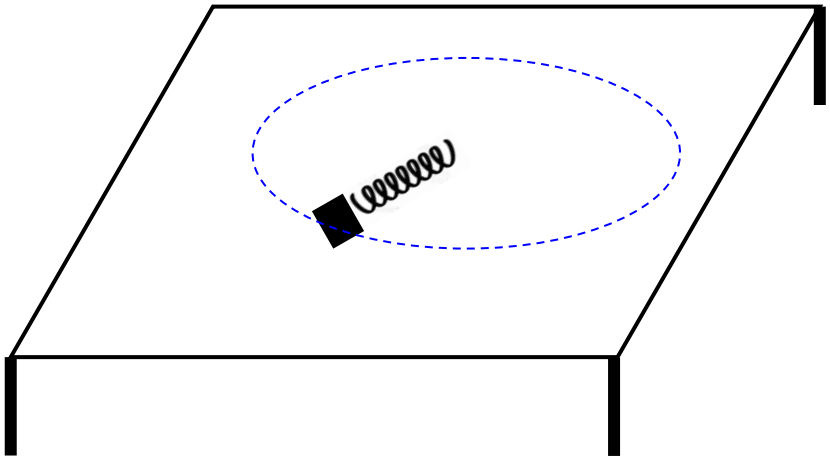
Teacher notes
Topic C

An example of resonance

A block of mass *m* is attached to a spring obeying Hooke's law with spring constant *k*. The block rotates on a horizontal frictionless table with angular frequency ω in a circle of radius *R*.



What happens when $\omega^2 = \frac{k}{m}$?

Let the extension of the spring be *e*. Then

$ke = m\omega^2 R$

The radius is $R = L + e$ where *L* is the natural unstretched length of the spring. This gives

$ke = m\omega^2 (L + e)$

And so

$$(k - m\omega^2)e = m\omega^2 L \Rightarrow e = \frac{m\omega^2 L}{k - m\omega^2}$$

We now see that if $\omega^2 = \frac{k}{m}$ the extension of the spring becomes infinite.

This is because the angular frequency $\omega^2 = \frac{k}{m}$ is the natural frequency of oscillation of the mass-spring system and we have a resonance condition.